GEOMETRY

Properties of lines

Intersecting Lines and Angles

If two lines intersect at a point, then opposite angles are called ***vertical angles*** and they have the same measure.

Perpendicular Lines

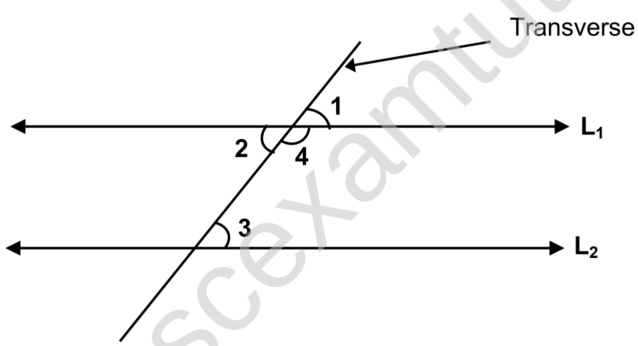
An angle that measures 90^o is a right angle. If two lines intersect at right angels, the lines are perpendicular to each other.

Parallel Lines

If two lines in the same plane do not intersect, they are parallel to each other.

Lines AB and CD are parallel and denoted by AB││CD.

Parallel lines and a transverse:



In the above given figure, the two lines L_1 & L_2 are parallel to each other and T is the transverse to both the lines.

Then we will have,

∠1 = ∠3 (Pair of corresponding angles)

∠2 = ∠3 (Pair of alternate angles) &

∠3 + ∠4 = 180^0 (Sum of interior angles)

Polygons:

A closed plane figure made up of several line segments that are joined together is called a polygon.

Types of Polygons

- **Equiangular** (All angles equal)
- **Equilateral** (All sides equal)
- **Regular** (All sides & angles equal)

Properties of Polygon:

1. Sum of all the exterior angles of any regular polygon is equal to 360^0.
2. Each exterior angle of an **n** sided regular polygon is $\frac{360^0}{N}$ degrees.
3. Each interior angle of an n sided equiangular polygon is $\frac{(n-2)x180^0}{n}$.
4. Also as each pair of interior angle & exterior angle is linear.
   Each interior angle = 180^o – exterior angle.
5. Area of a regular polygon = $\frac{1}{2}N.Sin\left(\frac{360}{n}\right)^0 xS^2$

   (N = Number of sides and S = length from center to a corner)
6. The sum of all the interior angles of n sided polygon is $(n – 2)180^0$

Triangles and Their Properties

On the basis of sides, triangles are classified into three categories

a) **Scalene:** Having all sides unequal.

b) **Isosceles:** Having any two sides of same length.

c) **Equilateral:** Having all the three sides of equal length.

On the basis of angles, triangles are divided into three categories:

a) **Obtuse angled triangle:** Largest angle greater than 90^0.

b) **Acute angled triangle:** All angles less than 90^0.

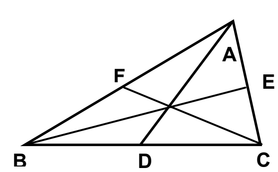
c) **Right Angled Triangle:** Largest angle equal to 90^0.

Properties of a Triangle:

1. Sum of the all the three angles is 180^o.
2. An exterior angle is equal to the sum of the interior opposite angles.
3. The sum of any two sides is always greater than the length of the third side.
4. The difference between any two sides is always less than that of the third side.
5. The side opposite to the greatest angle is the greatest side and the side opposite to the smallest angle is the shortest side.

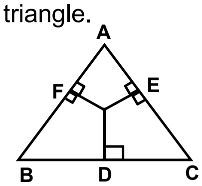
Points inside or outside a triangle with their properties:

Centroid: The point of intersection of the medians of a triangle



1. The centroid divides each median from the vertex in the ratio 2 : 1.
2. Apollonius theorem gives the length of the median.
   $AB^2 + AC^2 = 2(AD^2 + BD^2)$
3. If x, y, z are the lengths of the medians through A, B, C of a triangle ABC,
   $x^2 + y^2 + z^2 = (a^2 + b^2 + c^2)$.
4. Median always divides a triangle into two equal portions.

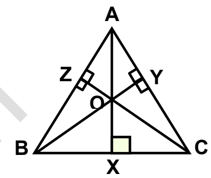
Circumcentre: The point of intersection of perpendicular bisectors of the sides of a triangle.



1. The circumcentre is equidistant from the vertices.
2. If a, b, c, are the sides of the triangle, Δ is the area & R is the radius of the circum-circle, then

   abc = 4R. Δ
3. In a right angled, the median to the hypotenuse is equal to its circumradius and is equal to half the hypotenuse.

Orthocentre: The point of intersection of the altitudes of a triangle.



1. B, Z, Y, C lie on a circle and form a cyclic quadrilateral.
2. C is the orthocentre of the right angled triangle ABC right angled at C.
3. Centroid divides the line joining the orthocentre and circumcentre in the ratio of 2 : 1.

Problem:

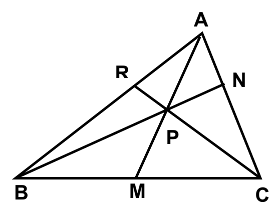
The orthocentre of a triangle is at (5, – 9) and the circumcentre is at (– 1, 4). Find the sum of x coordinates of all the three vertices of a triangle.

(1) 2 (2) 3 (3) – 2 (4) – 3

Hint: Centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1.

Incentre: The point of intersection of angle bisectors of the angles.

1. It is equidistant from the sides of the triangle.
2. According to Angle bisector theorem $\frac{BM}{MC} = \frac{AB}{AC}$



3. **Δ = rs,** if r is the radius of incircle, s = semi-perimeter and Δ is the area of the triangle.
4. $\frac{AP}{PM} = \frac{b+c}{a}$

Congruency of triangles:

Two triangles ABC and DEF are said to be congruent, if they are equal in all respects (equal in shape and size).

The notation for congruency is ≅ or ≡

If $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

AB = DE, BC = EF, AC = DF

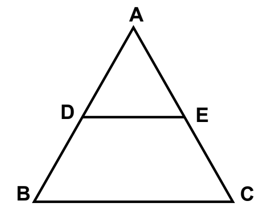
Then $\Delta ABC \equiv \Delta DEF$ or $\Delta ABC \cong \Delta DEF$

Mid Point Theorem:

A line joining the mid points of any two sides of a triangle is parallel and equal to half of the third side.

If in Δ ABC, D & E are the mid points of AB & AC respectively, then we have

$DE \parallel BC$ and $DE = \frac{1}{2}BC$



Problem

In ΔAEF, CD is parallel to EF. AD = DF, CD = 4 and DF = 3. What is EF?

(1) 3 (2) 4 (3) 8 (4) 6

Similar triangles:

Two figures are said to be similar, if they have the same shape but not the same size.

NOTE: Congruent triangles are similar but similar triangles need not be congruent.

Properties of similar triangles:

If two triangles are similar, the following properties hold true.

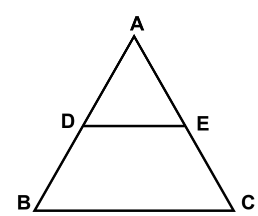
(a) The ratio of the medians is equal to the ratio of the corresponding sides.
(b) The ratio of the altitudes is equal to the ratio of the corresponding sides.
(c) The ratio of the internal bisectors is equal to the ratio of corresponding sides.
(d) The ratio of inradii is equal to the ratio of the corresponding sides.
(e) The ratio of the circumradii is equal to the ratio of the corresponding sides.
(f) Ratio of area is equal to the ratio of squares of the corresponding sides.
(g) Ratio of area is equal to the ratio of squares of the corresponding medians.
(h) Ratio of area is equal to the ratio of the squares of the corresponding altitudes.
(i) Ratio of area is equal to the ratio of the squares of the corresponding angle bisectors.

Basic Proportionality Theorem:

In a triangle, if a line drawn parallel to one side of a triangle divides the other two sides in the same ratio.

So if DE is drawn parallel to BC, it would divide sides AB and AC proportionally i.e.

$$\frac{AD}{BD} = \frac{AE}{EC}$$



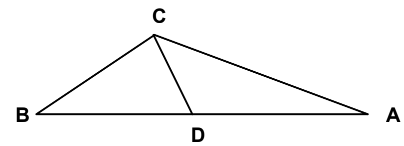
Pythagoras Theorem:

The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.

i.e. in a right angled triangle ABC, right angled at B,

$$AC^2 = AB^2 + BC^2$$

Angle Bisector Theorem:



If in ∆ABC, CD is the angle bisector of ∠BCA,
the ratio of the lines BD & AD is equal to the ratio of sides containing the angle.

$$\frac{BD}{AD} = \frac{BC}{AC}$$

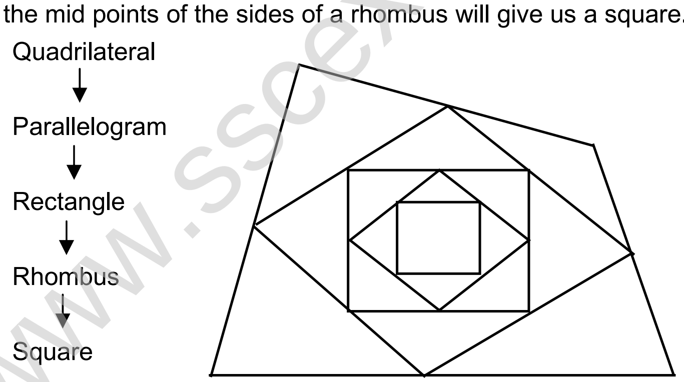
Quadrilateral (a four side closed figure):

Properties & Facts:

1. In a quadrilateral, sum of all four angles is equal to 360°.
2. The area of the quadrilateral = ½ × one diagonal x sum of the perpendicular to it from vertices.

Important Results

If we join the mid-points of the sides of a quadrilateral, we get a parallelogram and the mid-points of the sides of a parallelogram will give a rectangle. If we again join the mid-points of the sides of a rectangle, we get a rhombus and the mid points of the sides of a rhombus will give us a square.



Circles

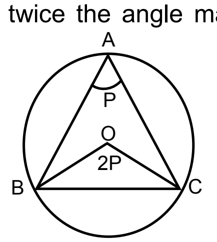
If O is a fixed point in a given plane, the set of points in the plane which are at equal distances from O will form a circle.

Properties of a Circle

1. If two chords of a circle are equal, their corresponding arcs have equal measure.
2. Measurement of an arc is the angle subtended at the centre. Equal arcs subtend equal angles at the center.
3. A line from centre and perpendicular to a chord bisects the chord.
4. Equal chords of a circle are equidistant from the centre.
5. When two circles touch, their centres and their point of contact are collinear.
6. If the two circles touch externally, the distance between their centres is equal to sum of their radii.
7. If the two circles touch internally, the distance between the centres is equal to difference of their radii.
8. Angle at the centre made by an arc is equal to twice the angle made by the arc at any point on the remaining part of the circumference.

   Let O be the centre of the circle.

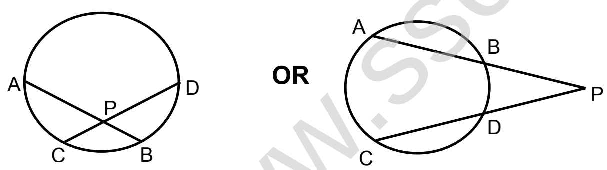
   $\angle BOC = 2\angle P$, when $\angle BAC = \angle P$



9. If two chords are equal, the arc containing the chords will also be equal.
10. The locus of the line joining the mid-points of all the equal chords of a circle is also a circle of radius, $\frac{1}{2}\sqrt{4r^2 - d^2}$ where r is the radius of the given circle and d is the length of equal chords.
11. There can be one and only one circle that touches three non-collinear points.
12. The angle inscribed in a semicircle is 90^o.

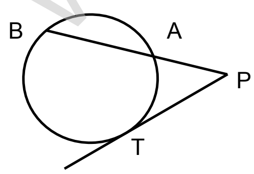
13. If two chords AB and CD intersect externally at P,

    $PA \times PB = PC \times PD$



14. If two chords AB and CD intersect internally at P,

    $PA \times PB = PC \times PD$

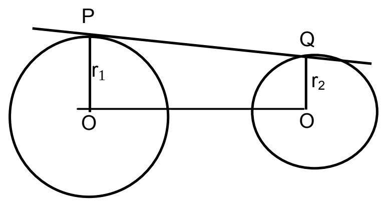


15. If PAB is a secant and PT is a tangent,

    $PT^2 = PA \times PB$

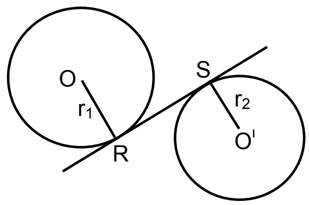
16. The length of the direct common tangent (PQ)

$= \sqrt{(\text{The distance between their centres})^2 - (r_1 - r_2)^2}$



17. The length of the transverse common tangent (RS)

$= \sqrt{(\text{The distance between their centres})^2 - (r_1 + r_2)^2}$



Cyclic Quadrilateral

If a quadrilateral is inscribed in a circle i.e. all the vertex lies on the circumference of the circle, it is said to be a cyclic quadrilateral.

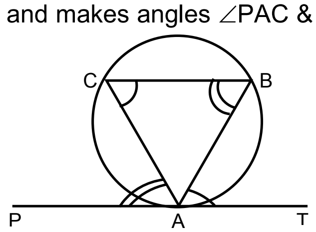
1. In a cyclic quadrilateral, opposite angles are supplementary.
2. In a cyclic quadrilateral, if any one side is extended, the exterior angle so formed is equal to the interior opposite angle.

Alternate angle theorem

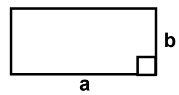
Angles in the alternate segments are equal.

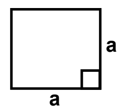
In the given figure, PAT is tangent to the circle and makes angles ∠PAC & ∠BAT respectively with the chords AB & AC.

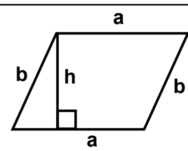
Then, BAT = ∠ACB & ∠ABC = ∠PAC

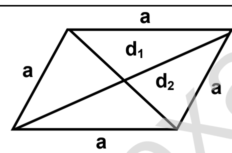


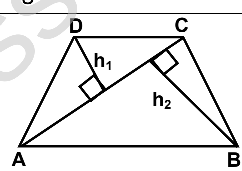
Formulae to calculate area of some geometrical figures:

<table>
<tr><th>S.No</th><th>Name</th><th>Figure</th><th>Perimeter in units of length</th><th>Area in square units</th></tr>
<tr><td>1.</td><td>Rectangle</td><td>

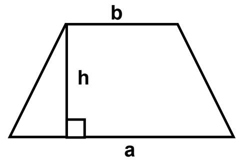
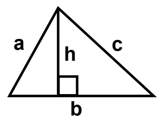
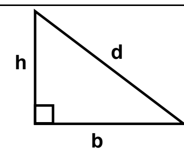
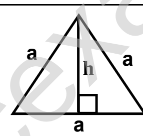
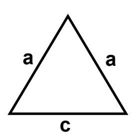
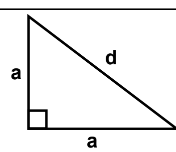
a = length
b = breadth</td><td>2(a + b)</td><td>ab</td></tr>
<tr><td>2.</td><td>Square</td><td>

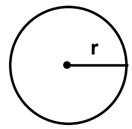
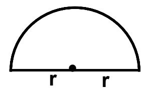
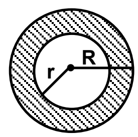
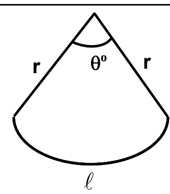
a = side</td><td>4a</td><td>a^2
$\frac{1}{2}$ (diagonal)^2</td></tr>
<tr><td>3.</td><td>Parallelogram</td><td>

a = side
b = side adjacent to a
h = distance between the opp. parallel sides</td><td>2(a + b)</td><td>ah</td></tr>
<tr><td>4.</td><td>Rhombus</td><td>

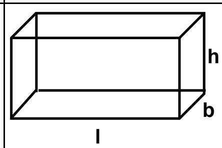
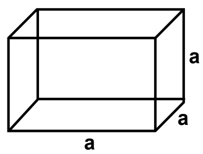
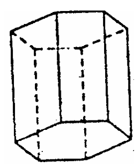
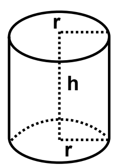
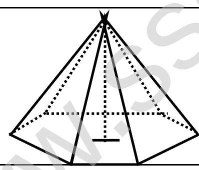
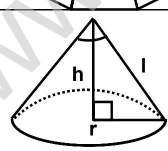
a = side of rhombus;
d_1,d_2 are the two diagonals</td><td>4a</td><td>$\frac{1}{2} d_1 d_2$</td></tr>
<tr><td>5</td><td>Quadrilateral</td><td>

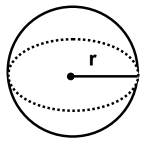
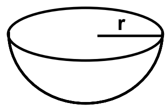
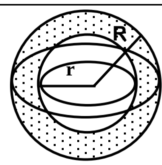
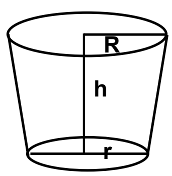
AC is one of its diagonals and h_1, h_2 are the altitudes on AC from D, B respectively.</td><td>Sum of its four sides</td><td>$\frac{1}{2}$ (AC) (h_1 + h_2)</td></tr>
</table>

6.	Trapezium	 a, b, are parallel sides and h is the distance between parallel sides	Sum of its four sides	$\frac{1}{2}h(a+b)$
7.	Triangle	 b is the base and h is the altitude. a, b, c are three sides of Δ.	a + b + c = 2s where s is the semi perimeter.	$\frac{1}{2}b \times h$ or $\sqrt{s(s-a)(s-b)(s-c)}$
8.	Right triangle	 d(hypotenuse) $= \sqrt{b^2+h^2}$	b + h + d	$\frac{1}{2}bh$
9.	Equilateral triangle	 a = side h = altitude = $\frac{\sqrt{3}}{2}a$	3a	(i) $\frac{1}{2}ah$ (ii) $\frac{\sqrt{3}}{4}a^2$
10.	Isosceles triangle	 c = unequal side a = equal side	2a + c	$\frac{c\sqrt{4a^2-c^2}}{4}$
11.	Isosceles right triangle	 d(hypotenuse) $= a\sqrt{2}$ a = Each of equal sides. The angles are 90^o, 45^o, 45^o.	2a + d	$\frac{1}{2}a^2$

12.	Circle	 r = radius of the circle $\pi = \frac{22}{7}$ or 3.1416	$2\pi r$	πr^2
13.	Semicircle	 r = radius of the circle	$\pi r + 2r$	$\frac{1}{2}\pi r^2$
14.	Ring (shaded region)	 R = outer radius r = inner radius	….	$\pi (R^2 - r^2)$
15.	Sector of a circle	 θ^o = central angle of the sector r = radius of the sector ℓ = length of the arc	$\ell + 2r$ where $\ell = \frac{\theta}{360} \times 2\pi r$	$\frac{\theta}{360} \times \pi r^2$

Volume of some solid figures

S. No	Nature of the solid	Shape of the solid	Lateral/ curved surface area	Total surface area	Volume	Abbreviations Used
1.	Cuboid		2h (l + b)	2(lb + bh + lh)	lbh	l = length b = breath h = height
2.	Cube		$4a^2$	$6a^2$	a^3	a = length of edge
3.	Right prism		(perimeter of base) × Height	2 (area of one end) + lateral surface area	Area of base × height	
4.	Right circular cylinder		$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$	r = radius of base h = height of the cylinder
5.	Right pyramid		$\frac{1}{2}$(Perimeter of the base) × (slant height)	Area of the base + lateral surface area	$\frac{1}{3}$(Area of base) × height	
6.	Right circular cone		$\pi r l$	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$	h = height r = radius l = slant height

S. No	Nature of the solid	Shape of the solid	Lateral/ curved surface area	Total surface area	Volume	Abbreviations Used
7.	Sphere		—	$4\pi r^2$	$\frac{4}{3}\pi r^3$	r = radius
8.	Hemi-sphere		$2\pi r^2$	$3\pi r^2$	$\left(\frac{2}{3}\pi r^3\right)$	r = radius
9.	Spherical shell		—	$4\pi (R^2 - r^2)$	$\frac{4}{3}\pi(R^3 - r^3)$	R = outer radius r = inner radius
10.	Volume of bucket				$\frac{\pi h}{3}(R^2 + r^2 + Rr)$	R = larger radius r = smaller radius h = height